where π is the product of n factors.

Now this likelihood function is to be maximised with respect to parameters α, β, σ^2. The partial differentiations when equated with zero give following equations.

$$\frac{\partial(\log L)}{\partial\alpha}=\frac{1}{\sigma^2}\,\Sigma\,(Y_i-\alpha-\beta X_i)=0$$

$$\frac{\partial(\log L)}{\partial\beta}=\frac{1}{\sigma^2}\,\Sigma\,[X_i\,(Y_i-\alpha-\beta X_i)]=0$$

$$\frac{\partial(\log L)}{\partial\sigma^2}=\frac{n}{2\sigma^2}+\frac{1}{2\sigma^4}\,\Sigma\,(Y_i-\alpha-\beta X_i)^2=0$$

On solving we get

$$\hat{\alpha}=\bar{Y}-\hat{\beta}\bar{X}$$

$$\hat{\beta}=\frac{\Sigma\,(X_i-\bar{X})\,(Y_i-\bar{Y})}{\Sigma\,(X_i-\bar{X})^2}$$

$$\sigma^2=\frac{\Sigma\,(Y_i-\hat{\alpha}-\hat{\beta}X_i)^2}{n}$$

Therefore both least squares estimators and maximum likelihood estimators of α & β are the same. Also like OLS estimators maximum likelihood estimators of $\hat{\alpha}$ & $\hat{\beta}$ are best linear unbiased estimators.

5·8. Standard Error

As we have proved that variance of $\hat{\beta}$ is $\frac{\sigma^2_u}{\Sigma x_i^2}$. However the variance of the true disturbances is itself unknown and must be estimated on the basis of the sample data. Once the regression line has been fitted, the residuals $(Y_i-\hat{Y}_i)$ can be computed and hence an estimate of σ^2_u can be ascertained—

as,

$$e_i=Y_i-\hat{Y}=y_i-\hat{y}_i$$

$$=\beta x_i+(u_i-\bar{u})-\hat{\beta}x_i$$

$$=-(\hat{\beta}-\beta)x_i+u_i-\bar{u}$$

$$e_i^2=(\hat{\beta}-\beta)^2x_i^2+(u_i-\bar{u})^2-2(\hat{\beta}-\beta)\,(u_i-\bar{u})x_i$$

taking sum of both sides

$$\Sigma e_i^2=(\hat{\beta}-\beta)^2\Sigma x_i^2+\Sigma(u_i-\bar{u})^2-2(\hat{\beta}-\beta)\Sigma x_i(u_i-\bar{u})$$

taking expectation of both sides

$$E(\Sigma e_i^2)=E[(\hat{\beta}-\beta)^2\Sigma x_i^2+\Sigma(u_i-\bar{u})^2-2(\hat{\beta}-\beta)\Sigma x_i(u_i-\bar{u})]$$

we have already proved

$$E(\hat{\beta}-\beta)^2\Sigma x_i^2=\sigma_u^2$$

$$E[\Sigma(u_i-\bar{u})^2]=E[\Sigma u_i^2-2\bar{u}\Sigma u_i+n\bar{u}^2]$$

$$=E\left[\Sigma u_i^2-\frac{1}{n}\left(\Sigma u_i\right)^2\right]=\left(n-1\right)\sigma_u^2$$

and $E(\beta-\hat{\beta})\Sigma x_i(u_i-\bar{u})=\sigma_u^2$

$$\therefore \quad E(\Sigma e_i^2)=\sigma_u^2+(n-1)\,\sigma_u^2-2\sigma_u^2$$

$$=\sigma_u^2\,(n-2)$$

$$S^2=\hat{\sigma}_u^2=\frac{\Sigma e_i^2}{n-2}$$

where S is the standard error.

as we know $\text{var.}\,(\hat{\beta})=\sigma^2_{\hat{\beta}}=\dfrac{\sigma_u^2}{\Sigma x_i^2}$

$$S^2_{\hat{\beta}}=\frac{S^2}{\Sigma x_i^2}=\frac{\Sigma e_i^2}{(n-2)\,\Sigma x_i^2}$$

or

$$S_{\hat{\beta}}=\sqrt{\left(\frac{\Sigma e_i^2}{(n-2)\Sigma x_i^2}\right)}$$

5·9. Hypothesis Testing

Till now we have discussed least squares estimation method. One of the prime objective of econometrics is to analyse data in a manner that allows us to test and evaluate the constructed model. An investigater may have several plausible models among which to choose. The choice no doubt depends upon purpose of the model and it also depends upon the availability of data. So each model must be specified in a form which yields empirically testable hypothesis. If data are consistent with the model, then model is implicitly accepted. Hence hypothesis testing constitutes an important part of the analysis and will result either in rejection or acceptance of the model. Under certain assumption about the error term in the assumed model, we have seen that each of these estimates can be presumed as these values drawn from a distribution centred on the true value of parameter, and its variance can thus be estimated. This information can be used to test the hypothesis about the relationship between X and Y.

A statistical hypothesis is an assumption about the frequency of a random variable in applied situation given a f